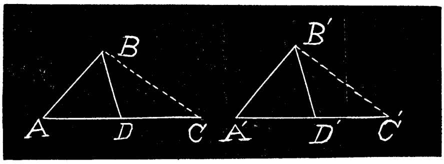
Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland, and CHAS. C. CROSS, Laytonsville, Maryland.

Let $AB=A'B'$, $AC=A'C'$, $BD=B'D'$. $\triangle ABD=\triangle A'B'D'$, because all the sides are equal, each to each.



Then $\triangle BDC=\triangle B'D'C'$, having two sides and included angle of one= two sides and included angle of the other.

$\therefore$ $\triangle ABC=\triangle A'B'C'$.

Also solved by *EDWARD R. ROBBINS, M. A. GRUBER,* and *G. B. M. ZERR.*

63. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Mississippi.

A rectangular hyperbola cannot be cut from a right circular cone *if the angle at its vertex is less than a right angle.*

Solution by the PROPOSER.

Let the base and the axis of the cone coincide with the xy-plane and the z-axis respectively. Then if c denote the altitude of the cone and ϕ the angle which any one of its elements makes with the base, its equation is

$$(x^2+y^2)\tan^2\phi=(z-c)^2.$$

The equation of a plane through the y-axis and inclined at an angle θ to the xy-plane is

$$z=x\tan\theta.$$

The projection on the xy-plane of the intersection of the two surfaces is

$$(x^2+y^2)\tan^2\phi=(x\tan\theta-c)^2=x^2\tan^2\theta-2cx\tan\theta+c^2.$$

This becomes, when referred to rectangular axes in the plane of the section, the origin and y-axis being unchanged, $(x^2\cos^2\theta+y^2)\tan^2\phi=x^2\sin^2\theta-2cx\sin\theta+c^2$, or $x^2(\cos^2\theta\tan^2\phi-\sin^2\theta)+y^2\tan^2\phi+2cx\sin\theta-c^2=0$, which represents a rectangular hyperbola if $\tan^2\phi+\cos^2\theta\tan^2\phi-\sin^2\theta=0$. From this equation,

$$\sin^2\theta=\frac{2\tan^2\phi}{\tan^2\phi+1}=2\sin^2\phi, \text{ and } \sin\theta=\pm\sqrt{2}\sin\phi.$$

Since $\sin\phi$ cannot be greater than $\frac{1}{\sqrt{2}}$, ϕ cannot exceed 45°. Hence the angle at the vertex of the cone cannot be less than 90°.

Other solutions of this problem will appear in next issue.

PROBLEMS.

67. Proposed by F. M. PRIEST, St. Louis, Mo.

Required: The length of a piece of carpet that is a yard wide with square ends, that can be placed diagonally in a room 40 feet long and 30 feet wide, the corners of the carpet just touching the walls of the room.

68. Proposed by LEONARD E. DICKSON, M. A., Ph. D., Formerly Fellow of Mathematics, University of Chicago; Chicago, Illinois.

Suppose a circle of unit radius divided at the points A, A_1, A_2, A_3, ... into n equal parts. [This division cannot in general be affected by geometry.] Through A draw the diameter OA and join O with A_1, A_2, A_3, ... $A_{\frac{n-1}{2}}$, where n is supposed to be odd.

Prove that $OA_1-OA_2+OA_3-OA_4+\ldots\pm OA_{\frac{n-1}{2}}$, every other chord being affected with the minus sign.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

36. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, New Windsor College, New Windsor, Maryland.

A vertical slit is made in the middle of the side of a rectangular box containing water. What is the time required to empty the box?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let a, b, h=length, width, and depth of box, c=width of slit, m=coefficient of contraction, z=distance of surface of water from bottom of box, x=distance of any elemental area of slit from bottom of box.

$\therefore$ The quantity discharged through the slit in an element of time is

$$Q=[mc\sqrt{2g}\int_0^z \sqrt{z-x}\,dx]dt=\tfrac{2}{3}mc\sqrt{2g}\,z^{\frac{3}{2}}\,dt=-abdz.$$

$$\therefore\ t=\frac{3ab}{2mc\sqrt{2g}}\int_{h'}^{h}\frac{dz}{z^{\frac{3}{2}}}=\frac{3ab(\sqrt{h}-\sqrt{h'})}{mc\sqrt{2ghh'}},\text{ for depth } (h-h').$$

When $h'=0$, t=infinity or it is impossible to absolutely empty the box.

II. Solution by the PROPOSER.

Let x=distance from base of box to any point in the vertical slit below surface of water.

Let y=distance from base of box to surface of water.

The velocity of discharge for point $x=\sqrt{2g(y-x)}$.